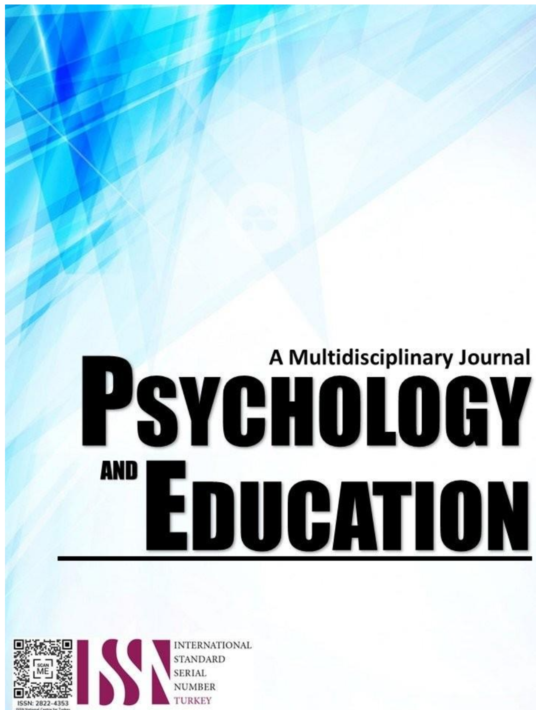
F-RED: FINDING THE AREA OF THE SHADED REGION BETWEEN THE CIRCLE AND TRIANGLE



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F-RED: Finding the Area of the Shaded Region Between the Circle and Triangle

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Abstract

Among all the geometric problems, determining the area of the shaded region between a circle and a triangle is frequently provided for both a circumscribed circle and an inscribed circle in a triangle are often given. Having limited time in competitive settings, calculating its area can be pretty time-consuming. Thus, the formula, referred to as F-RED (Formula RED), was derived to establish a consistent relationship between the area of the triangle to the inscribed circle and the circumscribing circle. With the help of the constant value (5.19615) that has been calculated, it is easier to get the area of the inscribed circle in a triangle with the given measurement of the radius, even without the help of other given numbers, by just multiplying it with radius squared and subtracted to the area of the circle. Area of the shaded region = (5.19615) (radius)2 - (area of a circle) Likewise, with the help of the constant value (1.299), it is easier to get the area of the circumscribing circle to a triangle using only the radius, even without the help of other given numbers, by just subtracting the area of the circle to the constant value multiplied by the radius squared. Area of the shaded region = (area of a circle) - (1.299) (radius)2 The researcher discovered that the area of the shaded region can now be determined just using the radius and constant values. Thus, all assumptions are proven correct.

Keywords: *shaded region, inscribed circle, circumscribed circle, geometric formula (F-RED), radius-based calculation*

Introduction

Among all geometric problems, finding the area of the shaded region of an inscribed circle in a triangle and a circumscribed circle to a triangle is often given. Calculating the area of the shaded region in an inscribed circle can be time consuming especially in competitive contexts. To determine this area, we need to calculate the areas of the circle and triangle using their respective formulas: π x radius2 and ½ x b x h. The conventional method requires step-by-step calculations, which can be burdensome during mathematical competitions where efficiency is crucial. The area of the shaded zone can be determined by subtracting the area of the circle from the area of the triangle, expressed as the Area of the shaded region = (½ x b x h) – (π x radius^2).

Furthermore, considering the height or the sides of the triangle, the primary challenge lies in the inability to immediately compute the area of the circle without first calculating its diameter by deriving the radius from the three sides of the triangle measured from the center. After getting the diameter of a circle, we can now start solving the areas of the two-plane figure using again the formula written above and the same process in determining the area of the shaded region of a circumscribing circle around a triangle, expressed as the area of the shaded region = (π x radius^2) – (½ x b x h).

Addressing these types of problems, especially in mathematical competitions, requires precision and time efficiency if one is aiming to win. In coherence, the researcher seeks to assist students, particularly those skilled in mathematics, in achieving these goals by developing a shortcut formula for finding the shaded regions of two-plane figures when inscribed or circumscribed.

Research Objectives

This study aimed:

1. To determine the constant value that can be utilized to create a simplified equation for finding the area of the shaded region between the circle and triangle of a circle inscribed in a triangle and for that circle circumscribing to a triangle.
2. To derive the shortcut formulas in finding the area of the shaded region between the circle and a triangle of an inscribed circle in a triangle.
3. To derive the shortcut formulas in finding the area of the shaded region between the circle and a triangle in a circumscribing circle to a triangle:

Methodology

Materials

- AutoCAD software
- Laptop
- Protractor
- Pen
- Paper
- Calculator

Procedure

Finding the shortcut formula for the area of shaded region between the circle and triangle of circumscribing circle to a triangle.

Likewise, the investigator assigned different radius lengths for a circle, ranging from 1 cm to 10 cm, and calculated the area of the shaded region of a circle inscribed within a triangle using the existing formulas: A=1/2bh for the area of a triangle, and A= πr^2 for the area of a circle; Compare the traditional formula and the formulas for finding the areas of the two plane figures to identify any shared characteristics; Solve for the constant value by determining the common difference for different radius lengths and simplify to obtain a constant number, then use this to formulate a new relation or formula for finding the area of the shaded region of an inscribed circle within a triangle, using only the circle's radius. Refer to Figure 1

Derivation for the shortcut formula in finding the shaded area between the triangle and circle of an inscribed circle in a triangle

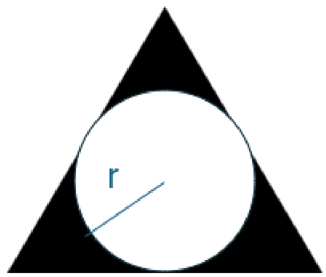

Figure 1. Inscribed Circle in a Triangle

Let the circle's radius be denoted as r.

Area of shaded region = bhx1/2 - πr^2

On the condition that the circle's radius is three times the triangle's height.

Table 1. *Parts of a triangle and the radius of the circle*

Radius (r)	*Triangle's Height*	*Triangle's Base*	*Triangle's Area*
1	3	3.4641	5.19615
2	6	6.9282	20.7846
3	9	10.3923	46.76535
4	12	13.8564	83.1384
5	15	17.3205	129.90375
Common Difference	3	3.4641	

Since the measurement of the height and base both rise in proportion to the length of the radius, we can now substitute both base and height as

Area of triangle = Base · Height /2

= (3.4641) r · (3) r /2

$$A_{\text{triangle}} = \frac{(3.4641)\cdot r \cdot (3)\cdot r}{2} \qquad A_{\text{triangle}} = \frac{(10.3923)\cdot r \cdot r}{2} \Rightarrow A_{\text{triangle}} = (5.19615)\cdot r \cdot r$$

$$A_{\text{triangle}} = (5.19615)\cdot r^2$$

Derivation of new formula:

Area of the shaded region = (constant)(radius)2 - (area of a circle)

In symbol: $A_{\text{shaded region}} = (5.19615)\, r^2 - \pi r^2$

Finding the shortcut formula of the area of the shaded region between the circle and triangle of the inscribed circle in a triangle.

The investigator assigned various radius lengths for a circle, from 1 cm to 10 cm, and computed the area of the shaded region of a circle inscribed within a triangle using the established formulas: A= b h × 1/2 for the area of a triangle, and A= πr^2 for the area of a circle; Analyze the traditional formula and the formulas for calculating the areas of the two plane figures to look for any shared properties;

Solve for the constant value by determining the common difference for different radius lengths and simplify to obtain a constant number, then use this to formulate a new relation or formula for finding the area of the shaded region of an inscribed circle within a triangle, using only the circle's radius. Refer to Figure 2

Derivation of the shortcut formula for calculating the shaded area between a triangle and its circumscribed circle.

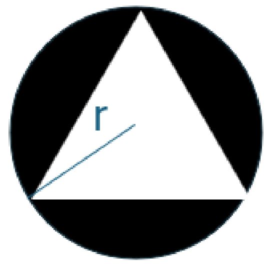

Figure 2. Circumscribing Circle to a Triangle

Let the circle's radius be denoted as r.

Area of shaded region = πr^2 - bhx1/2

Considering that the circle's radius is one and a half times the triangle's height.

Table 2. *Presentation of circle's radius and triangle's parts*

Radius (r)	*Triangle's Height*	*Triangle's Base*	*Triangle's Area*
1	1.5	1.7321	1.299075
2	3	3.4641	5.19615
3	4.5	5.1962	11.69145
4	6	6.9282	20.7846
5	7.5	8.6603	32.476125
Common Difference	1.5	1.732	

Since the measurement of the height and base both rise in proportion to the length of the radius, we can now substitute both base and height as

Area of shaded region = πr2 - Base · Height /2

= (1.732) r · (1.5) r/2

$$A = \frac{(1.732)r \cdot (1.5)r}{2} \qquad A = \frac{2.598 \cdot r \cdot r}{2} \qquad A = 1.299r^2$$

Derivation of new formula:

Area of the shaded region = (area of a circle) - (constant) (radius)2

In symbol: $A_{\text{shaded region}} = \pi r^2 - (1.299)\, r^2$

Testing and comparing both formulas

Tables 1 and 2 demonstrate the effectiveness of the newly introduced shortcut formula for calculating the area of the shaded region in both an inscribed and a circumscribed circle within a triangle.

Results and Discussion

Tabulated result in finding relation of Inscribe Circle in a Triangle given the length of an edge of a Triangle

Table 3. *(Inscribed circle in a triangle) Devised shortcut formula for finding the shaded area of an inscribed circle in a triangle*

Length of Radius	*Area of a Triangle*	*Area of a Circle*	*Shaded Area using Derived Formula*	*Constant Value to be multiplied to the Area of Circle*	*Shaded Area using Derived Formula $A = -(5.19615)\, r^2 - \pi r^2$*	*Remarks*
1 cm	5.19615 cm^2	3.14 cm^2	2.05615 cm^2	5.19615	2.05615 cm^2	Exactly the same
2 cm	20.7846 cm^2	12.56 cm^2	8.2246 cm^2	5.19615	8.2246 cm^2	Exactly the same
3 cm	46.76535 cm^2	28.26 cm^2	18.50535 cm^2	5.19615	18.50535 cm^2	Exactly the same

4 cm	83.1384 cm^2	50.24 cm^2	32. 8984 cm^2	5.19615	32. 8984 cm^2	Exactly the same
5 cm	129.90375 cm^2	78.5 cm^2	51.40375 cm^2	5.19615	51.40375 cm^2	Exactly the same
6 cm	187.0614 cm^2	113.04 cm^2	74.0214 cm^2	5.19615	74.0214 cm^2	Exactly the same
7 cm	254.61135 cm^2	153.86 cm^2	100.75135 cm^2	5.19615	100.75135 cm^2	Exactly the same
8 cm	332.5536 cm^2	200.96 cm^2	131.5936 cm^2	5.19615	131.5936 cm^2	Exactly the same
9 cm	420.88815 cm^2	254.34 cm^2	166.54815 cm^2	5.19615	166.54815 cm^2	Exactly the same
10 cm	519.615 cm	314 cm	205.615 cm	5.19615	205.615 cm	Exactly the same

Table 3 reveals that the constant value of 5.19615 when multiplied by the radius squared and subtracted from the circle's area, gave exactly the value using the traditional method (5.19615 r2 - πr2). The investigator confirmed the accuracy of the table and determined that the newly developed shortcut formula for calculating the shaded area of an inscribed circle in a triangle yields the same result as the traditional method.

Table 4. *(Circumscribing circle of a triangle) Devised shortcut formula for determining the shaded area of a circumscribed circle around a triangle*

Length of Radius	*Area of a Triangle*	*Area of a Circle*	*Shaded Area using Derived Formula*	*Constant Value to be multiplied to the Area of Circle*	*Shaded Area using Derived Formula A = πr2- (1.299) r2*	*Remarks (both round off to the hundredths)*
1 cm	1.299075 cm2	3.14 cm2	1.840925 cm2	1.299	1.841 cm2	Exactly the same
2 cm	5.19615 cm2	12.56 cm2	7.36385 cm2	1.299	7.364 cm2	Exactly the same
3 cm	11.69145 cm2	28.26 cm2	16.56855 cm2	1.299	16.569 cm2	Exactly the same
4 cm	20.7846 cm2	50.24 cm2	29.4554 cm2	1.299	29.456 cm2	Exactly the same
5 cm	32.476125 cm2	78.5 cm2	46.023875 cm2	1.299	46.025 cm2	+0.01
6 cm	46.76535 cm2	113.04 cm2	66.27465 cm2	1.299	66.276 cm2	+0.01
7 cm	63.6531 cm2	153.86 cm2	90.2069 cm2	1.299	90.209 cm	Exactly the same
8 cm	83.1384 cm2	200.96 cm2	117.8216 cm2	1.299	117.824 cm	Exactly the same
9 cm	105.222375 cm2	254.34 cm2	149.117625 cm2	1.299	149.121 cm	Exactly the same
10 cm	129.90375 cm2	314 cm	184.09625 cm2	1.299	184.1 cm	Exactly the same

Table 4 reveals that the circle's area is subtracted to the constant value of 1.299 when multiplied by radius squared (πr2 - 1.299r2) gave an approximate value using the traditional method when it is rounded off to the nearest hundredths with radii 5 and 6 having only 0.01 marginal error. The investigator confirmed the table and found that the newly developed shortcut formula for calculating the shaded area of the circumscribed circle around a triangle yields the same result as the traditional method when rounded to the nearest tenth.

Conclusions

Based on the findings, F-RED: Finding the Shaded Region Between the Circle and Triangle yields a positive result in terms of comparing both methods. The measurement of the height and base both rise in proportion to values the length of the radius provided; that way, the investigator can simplify the formula to get the constant value and get every problem revolving around this topic using only the radius. The researcher, therefore, concludes that with the two constant (5.19615) and (1.299) that have been calculated, we can easily get the area shaded region between the two planes.

In terms of proving the shaded region between the circle and the triangle of an inscribed circle in a triangle (5.19615 r2 - πr2), getting the exact value of the shaded region area is much easier than using the usual method.

In terms of proving the shaded region between the circle and triangle of a circumscribing circle to a triangle (πr2 - 1.299r2), it is approximately equal to the exact value of the shaded region area using the traditional method when both methods are rounded by hundredths with a radius 5 and 6 having a marginal error of 0.01 while both methods are exactly equal when it is rounded in the nearest tenths.

Based on the research findings and conclusions, further studies should be conducted to validate whether the newly discovered formulas can be applied to other plane figures that are also inscribed and circumscribed.

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